Appendix J

List of Symbols

Chapter 1.	**Introduction**	
%	Percent, 1/100	1
‰	Permille, 1/1000	1
ppm	Parts per million, 10^{-6}	4
Chapter 2.	**Origins**	
τ	Turn	18, 63
°	Degree, 1/360 turn	18,198
′	Arcminute, 1/60 degree	18
″	Arcsecond, 1/60 arcminute	18
$\lfloor x \rfloor$	Floor function	22
$\lceil x \rceil$	Ceiling function	22
$p \bmod q$	Modulo operator	22
$(x)_b$	Number in base b: $(\ldots d_1 d_0 . d_{-1} d_{-2} \ldots)_b = \sum_{k \in \mathbb{Z}} d_k b^k$	22
$\sum_{k=a}^{b} f(k)$	Summation symbol	23, 90
$\int_a^b f(x)dx$	Integration symbol	23
$\lim_{n \to \infty} f(n)$	Limes symbol	23
$\uparrow^n$	Knuth's up-arrow notation	29
$\sqrt{\ }$	Square root	31, 198
△ABC	Triangle with vertices A, B and C	42
∼	Similarity of geometrical objects	42
$[a_0; a_1, \ldots]$	Continued fraction	60, C1
π	Pi	63, 199
≅	Congruence of geometrical objects	64
Chapter 3.	**Basics**	
∧	And, Meet	76, 80
∨	Or, Join	76, 80
¬	Not, Complement	76, 80
→	If … then…, logical conditional	76
≡ , ≔	Identity, Definition	77
↔	If and only if, logical biconditional	77
$\vdash$, $\vdash_{\mathcal{FS}}$	Syntactic consequence in formal system $\mathcal{FS}$	78
$\models$, $\models_{\mathcal{FS}}$	Semantic consequence in formal system $\mathcal{FS}$	78
⊥	Bottom / Least / False	80
⊤	Top / Greatest / True	80
≤	Order relation	81

$\{\ \}$	Set	83
$\{:\}, \{\mid\}$	Set builder notation	83
$\in, \ni, \notin, \not\ni$	Membership and non-membership of set	83
$=, \neq$	Equality and non-equality	83
$\subseteq, \supseteq, \nsubseteq, \nsupseteq$	Subset and non-subset	83
$\subset, \supset, \not\subset, \not\supset$	Proper subset and non-proper subset	83
$\emptyset$	Empty set	83
$\mathbb{U}$	Universal set, containing all considered elements	83
$\cup$	Union	83
$\cap$	Intersection	83
$\sqcup$	Disjoint union	83
$\setminus, -$	Subtraction between sets	83
$\complement, \ ^c, \ ', \ ^-$	Complement of set	83
Δ	Symmetric difference	83
$\times$	Cartesian product	83
$\mathcal{P}(S)$	Power set of set S	83
$\mid\ \mid, \#$	Cardinality	83
$\mathbb{N}$, $\mathbb{N}_1$, $\mathbb{Z}^+$	Natural numbers {1,2,3 ... }	84
$\mathbb{N}_0$	Natural numbers starting from 0, {0,1,2, ... }	84
$\mathbb{Z}^-$	Negative numbers {-1, -2, -3, ... }	84
$\mathbb{Z}$	Integers $\{0, \pm 1, \pm 2, \dots\}$	84
$\mathbb{Q}$	Rational numbers	84
$\mathbb{A}$	Algebraic numbers	84
$\mathbb{R}$	Real numbers	84
$\mathbb{C}$	Complex numbers	84
$\mathbb{H}$	Quaternions	84
s.t. , : , $\mid$	Such that	84
$\Longrightarrow$	Implication	84
$\Leftrightarrow$, iff	Equivalence, if and only if	84
$\forall$	Universal quantifier, for all	84
$\exists$	Existential quantifier, there exists	84
$\exists!$	There exists exactly one	84
$\therefore$	Therefore	84
$(x_1, x_2, \dots, x_n)$	Ordered n-tuple	84
Q.E.D.,□, ■	Which should be proved	84
$\cdots, \vdots, \ddots, \therefore$	Ellipsis, continuing a given pattern	84
$f: X \to Y$	Function from set X to set Y	85
$x \mapsto y$	Mapping element x into element y	85
$f(x)$	Function value	85
$A \ni x \to y \in B$	Fully specified function	85
$\prod_{i=1}^{n} a_i$	Product symbol	90
$p_1 \to \cdots \to p_n$	Conway chained arrow notation	91
$\zeta(s)$	Riemann zeta function	92
$\sim$	Relation operator, $x \sim y$	94
$[x]$	Equivalence class of x	94

$\circ$	Functional composition	98
$\aleph_n$	Aleph n, Cardinality measure of set size	101
$\mathfrak{c}$	Cardinality measure of real numbers	101
$n^{\underline{k}}$	Falling factorial	102
$n^{\overline{k}}$	Rising factorial	102
$\binom{n}{k}, C(n,k), nCk$	Binomial coefficient	103
$\binom{n}{k_1, k_2, \ldots, k_m}$	Multinomial coefficient	103
$\Gamma(z)$	Gamma function	103
$\begin{bmatrix} n \\ k \end{bmatrix}$	Stirling numbers of the first kind	104
$\begin{Bmatrix} n \\ k \end{Bmatrix}$	Stirling numbers of the second kind	104
$\widehat{\boldsymbol{a}}$, $\langle a_n \rangle$	Sequence	107
Δ	Difference operator	108
$I\langle x_n \rangle$	Identity operator	108
$E\langle x_n \rangle$	Forward shift operator	108
$\star$	Convolution operator	111
χ	Euler characteristic	127
χ	Chromatic number	133
$\approx$	Approximation	135
i	Imaginary unit	137
Re, $\Re$	Real part	137
Im, $\Im$	Imaginary part	137
$\bar{z}$	Conjugate value	137
$\lvert \quad \rvert$	Absolute value, norm of complex number	137
$\sqrt[n]{\quad}$	n-th root	138, 198
Δ	Discriminant	139
$f \mid g$	Divisor	141
(f, g), GCD	Greatest common divisor	141
$\angle$	Angle	145
$\parallel$	Parallel lines	145
$\sim$	Similarity of geometrical objects	146
$\overline{\mathrm{AB}}$	Segment	148
$\overrightarrow{\mathrm{AB}}$, $\boldsymbol{v}$, $\vec{v}$	Vector notation	148
$\cdot$, $\langle \cdot, \cdot \rangle$	Scalar product, dot product	148
$\lVert \quad \rVert$	Length of vector, norm in vector space	148, 149
$(a_1, a_2, \ldots, a_n)$	n-tuple, finite ordered list, coordinate list	150
$\mathbb{E}^n$	Euclidean space	152
$\times$	Vector product	156
$\varepsilon_{i_1 i_2 \ldots i_n}$	Levi-Civita symbol	157
$\mathbb{RP}^n$, $\mathbb{CP}^n$	Projective spaces	160
$[a_1 : a_2 : \ldots : a_n]$	Homogeneous coordinates	160
$\langle a : b : c \rangle$	Trilinear coordinates	162

$\lim_{x \to x_0} f(x)$	Limit	165
$[a, b]$	Closed interval including endpoints	165
(a,b),]a,∞[	Open interval excluding endpoints	165
[a,b),]−∞,b]	Half-open intervals	165
$B_r(p)$, $B_r[p]$	open ( ) and closed [ ] balls around point p	165
S^n	n-sphere, surface of ball	165
$\text{int}(S)$, S^0	Interior of set	165
$\text{cl}(S)$, $\bar{S}$	Closure of set	165
∂S	Boundary of set	165
sup(S), $\sup_{x \in S} f(x)$	Supremum, least upper bound	170
inf(S), $\inf_{x \in S} f(x)$	Infimum, greatest lower bound	170
max(S), min(S)	Maximum and minimum taken from finite set	170
Δx	Delta, change or difference in a value/function	174
$f', Df, \dot{y}, \frac{dy}{dx}$	Derivative	174
$f'', D^2 f, \ddot{y}, \frac{d^2 y}{dx^2}$	Second derivative	174
$f^n, D^n f, \frac{d^n y}{dx^n}$	n-th derivative	174
$\frac{dy}{dx}\Big\|_{x=x_0}, \frac{dy}{dx}(x_0)$	Derivative at given point in Leibniz notation	174
$f_x, \partial_x f, D_x f, \frac{\partial f}{\partial x}$	Partial derivative	178’
$f_{xy}, \partial_{yx} f, \frac{\partial^2 f}{\partial y \partial x}$	Partial derivative	178’
$\frac{\partial^{\|\alpha\|} f}{\partial x^\alpha}, \frac{\partial^{\|\alpha\|} f}{\partial x_1{}^{\alpha_1} \ldots \partial x_n{}^{\alpha_n}}$	Partial derivative	178’
$C^0, C^0(M)$	Class of continuous functions (on set M)	168
$C^k, C^k(M)$	Continuously differentiable k times	175
$C^\infty, C^\infty(M)$	Continuously differentiable any number of times	175
$df, df(x.\Delta x)$	Differential	178
$\int_a^b f(x)dx$	Integral	179, 185
$\int f(x)dx\,, F(x)$	Antiderivative	179
$[F(x)]_a^b$	$F(b) - F(a)$	179
$\Delta = \{x_0, x_1, \ldots, x_n\}$	Partition of interval	185
$\|\Delta\|$	Norm of partition	185
$I_A(x)$	Indicator function of set A	186
$\int_\alpha^\beta f(x)d\alpha\,(x)$	Riemann-Stieltjes integral	189
$\ln(x)$	Natural logarithm function	193
e	Euler’s number	194
$\lg(x)$	Logarithm function of base 10	195
$\log_b(x)$	Logarithm of base b	195
$\exp_b(x)$	Exponential function of base b	195
x^y	Exponentiation	195